

## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Round 1  
Algebra I:  
Fractions and  
Exponents

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

- 1) Express as an integer or a reduced fraction:

$$(100)^2(50)^{-3}(25)^{-4}(10)^5(5)^6(2)^{-7}$$

- 2) Simplify as much as much as possible. Write your coefficient as a decimal number multiplied or divided by powers of variables using no negative exponents:

$$(10a^3b^4)^{-6} \left[ (5a)^3 (2b^{-4})^2 \right]^3$$

- 3) Perform the operations and simplify as much as possible, given that  $k \neq 0$ ,  $m \neq 0$ , and  $m \neq k$  or  $m \neq -k$ . Express as a single fraction with no negative exponents.

$$\frac{\frac{m^2 - k^2}{k^5 m^3} - \frac{k^2 - m^2}{k^3 m^5}}{m^2 + k^2}$$

## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Round 2 Algebra I: Fractional Expressions and Equations
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1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

1). Express as the quotient of two simplified polynomials. Write the terms of your polynomials in order of decreasing degree.

$$1 - \frac{1}{1 - \frac{2}{1 + \frac{3}{x - 4}}}$$

2). Solve for all possible values of x:

$$\frac{5x}{x - 5} + \frac{4}{x + 6} = \frac{54x + 5}{x^2 + x - 30}$$

3). Add and subtract these fractions and simplify your answer as much as possible. Assume that no values of r will make any expressions in the problem equal to zero. (Note: the first sign between fractions is division and the second sign is addition).

$$\frac{4r^2 - 1}{2r^2 - 9r - 5} \div \frac{2r^2 - 21r + 10}{3r^2 - 31r + 10} + \frac{4r^2 - 4r + 1}{r^2 - 10r + 25}$$

**FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013**

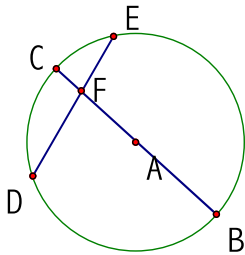
Match 5 Round 3  
Geometry:  
Circles

1.) \_\_\_\_\_

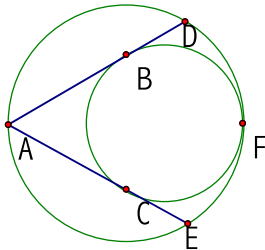
2.) \_\_\_\_\_

3.) \_\_\_\_\_

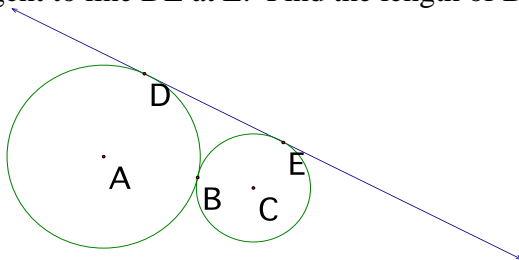
- 1). A circle of radius 9 has four points B,C,D,E that are on the circle. DE and BC intersect at point F in the interior of the circle. EF=7 and DE=15. If BC is a diameter of the circle, what two numbers give the lengths of BF and CF?



- 2) Two circles are internally tangent at point F. The radius of the smaller circle is  $\frac{2}{3}$  of the radius of the larger circle. Point A is on the larger circle. One segment drawn from point A is tangent to the smaller circle at B and intersects the larger circle at D. Another segment drawn from point A is tangent to the smaller circle at C and intersects the larger circle at E. Find the sum of the degree measures of minor arc BC and minor arc DE.



- 3) A circle with center A has radius 8 cm. A circle with center C has radius 5 cm. The two circles are externally tangent at B. Circle A is tangent to line DE at D and circle C is tangent to line DE at E. Find the length of DE.



## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Round 4

Quadratic  
Equations and  
Complex  
Numbers

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

1) A quadratic equation with integer coefficients has  $13 - i\sqrt{11}$  as one of its zeros. Find the quadratic equation expressed in the form  $ax^2 + bx + c = 0$  with  $a > 0$  and  $a, b,$  and  $c$  relatively prime integers.

2) Find all complex solutions of  $x^2 - (4 + 2i)x + 4i = -19$ .

3) Express the following as a complex number  $a + bi$  for real numbers  $a$  and  $b$ :

$$\frac{4i}{(1+i)^3} - \frac{2}{(1-i)^2} + \frac{3+4i}{2+i}$$

## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Round 5  
Solving Trig  
Equations

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

1) Solve for all  $x$  if  $0 \leq x < 2\pi$ :  $\sec^3 x - \sec x - \tan^2 x = 0$

2) Solve for all  $x$  if  $0 \leq x < 2\pi$ :  $\sin(2x) - \cos(x) = \cot(x)$

3) If  $\sin(A+B) - \sin(A-B) = \frac{-1}{16}$  and  $\cos A = \sin B + \frac{3}{4}$ , find all possible values of  $\cos A$ . Express your answers in simplest radical form.

## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Round 6 Sequences and Series
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1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

- 1) An infinite geometric series converges to 2500. A second infinite geometric series has the same common ratio, but the first term of the second series is 6 greater than the first term of the first series. The second series converges to 4000. What is the common ratio for each series?
  
  
  
  
  
  
  
  
  
  
- 2) In a geometric sequence, the fifth term is 16 and the ninth term is 2. If all terms are real numbers, give all possible values for the fourteenth term. Express your answers in simplest radical form.
  
  
  
  
  
  
  
  
  
  
- 3) The fourth term of an arithmetic sequence is 9.2. The seventh term of the sequence is 16.7. The sum of the first  $n$  terms is 288. Find  $n$ .

## FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 5 Team Round
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1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

- 1) If  $m = a^3b^2c^{-5}$  and  $n = a^2b^{-3}c^4$  and no variables are equal to zero, simplify  $\frac{7c^{22}m^2 - 3b^{13}n^3}{m^2n^3}$ . Express your answer with no negative exponents.

- 2) If  $x$  is equal to the infinite continued fraction  $2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$ , find the value of

$x$  in simplest radical form.

- 3) Each of three circles is tangent externally to the other two circles. The triangle formed by connecting the centers of the three circles is the smallest right triangle such that the radii of all the circles are integers. If  $x$  is the area of the smallest circle,  $y$  is the area of the middle-sized circle, and  $z$  is the area of the largest circle, give the ratio  $x:y:z$ .

- 4) Find the two complex square roots of  $33 - 56i$

- 5) If  $3 \cos x + 2 \sin x = 1$ , find all possible values of  $\cos x$ . Express your answers in simplest radical form.

- 6) In an arithmetic sequence, the 17<sup>th</sup> term is the square of the 5<sup>th</sup> term. If the 11<sup>th</sup> term is 6, what are all possible values for the first term of the sequence?